Ultimate Transmission Capacity of Amplified Optical Fiber Communication Systems taking into Account Fiber Nonlinearities

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Abstract

An analytical expression for the maximum achievable transmission capacity of multi-channel optical fiber communication systems with optical amplifier cascades is presented, taking into consideration amplifier noise and fiber nonlinearities.

Introduction

The main advantage and fascination of optical fiber transmission consists in the - in principle - abundance of bandwidth. The two physical factors which prevented from assessing this vast capacity in traditional systems were the linear fiber damping and the fiber dispersion. However, with the advent of erbium doped fiber amplifiers (EDFAs) in the late 80's the (3R-)repeater spacing could be increased to over thousands of kilometers [1]. Furthermore the detrimental impact of linear fiber dispersion on system performance could be considerably reduced by the the use of dispersion shifted fibers, for which even the second order group velocity dispersion can be controlled below 0.08 ps/(km $\cdot$nm^2). Alternatively, dispersion cancellation schemes using optical fiber equalizers (OFEs) have been successfully demonstrated [2, 3]. Latter compensate for the dispersion accumulated along a certain distance by adding a specifically designed fiber with the negative amount of dispersion, which can be produced with todays technology with attenuation coefficients comparable to standard single mode fibers, but having a dispersion of up to -45 ps/(nm$\cdot$km).
From this situation new limitations arise resulting on one hand from the optical amplifier noise present in systems incorporating optical amplifier-cascades and on the other hand from the nonlinear behaviour of the optical fiber. One way to handle those nonlinear effects, is to use the sophisticated soliton transmission [4]. The technical implementation of high bitrate soliton systems, however, is very complicated since definite input pulses have to be generated and a constant dispersion along the fibre must be assured. It is questionable whether the entire fiber bandwidth will be assessed by single channel soliton transmission. Although first soliton multichannel optical frequency division multiplexed systems have been demonstrated, several effects are expected to limit the performance of those schemes, among which the nonreciprocity of soliton-interaction occuring when using discrete amplifiers and the jitter due to spontaneous emission must be taken into account [5].
In our analysis we consider non-soliton transmission and the nonlinear effects are treated in the frequency domain as Four-Photon-Mixing. Using a statistical analysis the influence of the nonlinear distortions appears as noise similar to the amplifier noise. The optimum system performance is obtained by maximizing the signal-to-noise ratio (SNR). In contrast to solitons no compensation of linear and nonlinear effects is assumed. The calculations are carried out for one polarization state but apply as well to polarization mixing in standard fibers without major changes.

1 Accumulation of amplifier noise

In a transmission system of total length L, the loss caused by fiber attenuation is compensated for by optical amplifiers spaced by Δ_{OA}. The optical amplifiers can be modeled as two level systems [6] accumulating spontaneous emission noise (ASE) and resulting in the spectral noise density $Y(\omega)$ at the receiver:

$$Y(\omega) = \frac{1}{2}\hbar\omega + \frac{L}{\Delta_{OA}}\left[\exp(\alpha\Delta_{OA}) - 1\right] n_{sp}\hbar\omega \tag{1}$$

which takes its minimum for short amplifier spacings ($\alpha\Delta_{OA} \ll 1$) yielding for long distances ($\alpha L \gg 1$) simply

$$Y(\omega) = \alpha L n_{sp}\hbar\omega. \tag{2}$$

2 Nonlinear distortions

Using the nonlinear wave equation [7] for complex amplitudes $A(z,\omega)$ in the frequency domain

$$j\frac{\partial}{\partial z}A(z,\omega) = \frac{\beta_2}{2}(\omega-\omega_0)^2 A_\mu(z) + \gamma\left[A(z,\omega) * A(z,\omega) * A^*(z,-\omega)\right]. \tag{3}$$

with the dispersion term $\beta_2 = d^2\beta/d\omega^2$ and the nonlinearity coefficient γ we perform a statistical analysis on the propagation of these complex amplitudes $A(z,\omega)$. The basic assumption in this analysis is a signal source with uncorrelated spectral components in order to achieve a high transmission capacity. This simplifies the study of the distortions caused by Four-Photon-Mixing, which we can describe by the spectral noise density $X(\omega)$, which is given for uncorrelated amplitudes $A(0,\omega)$ with its spectral power density $W(\omega)$ as follows:

$$X(\omega) = 2\gamma^2\left(\frac{1}{2\pi}\right)^2 \iint W(\omega-\omega')W(\omega''-\omega')W(-\omega')\frac{4\sin^2\left(\beta_2(-\omega')(\omega''-\omega'-\omega)L\right)}{(\beta_2(-\omega')(\omega''-\omega'-\omega))^2}d\omega' d\omega'' \tag{4}$$

3 Discussion

For a signal with a constant spectral density $W(\omega) = W$ within a bandwith B we derive a coarse approximation for the noise spectral density $X(\omega_0)$ in the center ω_0 of the band by using

$$\int_{-\frac{b}{2}}^{\frac{b}{2}}\int_{-\frac{b}{2}}^{\frac{b}{2}} \frac{\sin^2(xy)}{(xy)^2}dx\,dy \approx \begin{cases} b^2 & \text{for } b^2 \le 2\pi \quad \text{low dispersion} \\ 2\pi\left(1+\ln\left(\frac{b^2}{2\pi}\right)\right) & \text{for } b^2 > 2\pi \quad \text{high dispersion} \end{cases} \tag{5}$$

With $x = (\omega-\omega'')\sqrt{|\beta_2|L}$, $y = (\omega''-\omega')\sqrt{|\beta_2|L}$ and $b = 2\pi B\sqrt{|\beta_2|L}$ we obtain for high dispersion ($2\pi B^2|\beta_2|L > 1$):

$$X(\omega_0) = \frac{4\gamma^2 W^3 L}{\pi|\beta_2|}\left(1+\ln\left(|\beta_2|L 2\pi B^2\right)\right) \tag{6}$$

yielding for the total signal-to-noise ratio SNR

$$\mathrm{SNR} = \frac{W}{Y+X} = \frac{W}{\alpha L n_{sp}\hbar\omega + \frac{4\gamma^2 W^3 L}{\pi|\beta_2|}\left(1+\ln\left(|\beta_2|L2\pi B^2\right)\right)} \tag{7}$$

In the system design the input power P and the spectral density W can be chosen in order to obtain a maximum $\mathrm{SNR_{opt}}$ for a certain $W = W_{opt}$.
For example, with an ideal optical amplifier ($n_{sp} = 1$) and a standard optical fiber with

$\omega_0 = 1.216 \cdot 10^{15}\frac{1}{s}$ ($\lambda = 1.55\mu$m)
$\gamma = 2\cdot 10^{-3}\frac{1}{\mathrm{Wm}}$ $\left(\chi^{(3)} = 2.3\times 10^{-22}\frac{\mathrm{m}^2}{\mathrm{V}^2},\ A_{\mathrm{eff}} = 44\mu\mathrm{m}^2\right)$
$L = 10^7$m (10 000km)

$\alpha = 4.6\cdot 10^{-5}\frac{1}{\mathrm{m}}$ $(0.2\frac{\mathrm{dB}}{\mathrm{km}})$
$\beta_2 = -20\frac{\mathrm{ps}^2}{\mathrm{km}}$ $(D = 15.7\frac{\mathrm{ps}}{\mathrm{nm\cdot km}})$
$B = 3750$GHz ($\Delta\lambda = 30$nm, EDFA)

we obtain

$W_{opt} = 8.684\cdot 10^{-16}\frac{\mathrm{W}}{\mathrm{Hz}}$ $\quad P_{opt} = B\cdot W_{opt} = 3.26$mW $\quad \mathrm{SNR_{opt}} = 9.8$

For a standard fiber with high dispersion, the full bandwidth of an Erbium doped fiber amplifier can thus be exploited with still a reasonable SNR.
The ultimate channel capacity C according to Shannon [8] is then given as $C = B \cdot \log_2(1 + \text{SNR}_{\text{opt}}) = 12.88\text{Tbit/s}$. For a fiber with high dispersion the capacity limit is thus set by very high limits, provided that a suitable dispersion compensation at the receiver is carried out. The Shannon limit for the transmission capacity is shown in Figures 1 - 3 for several parameters.

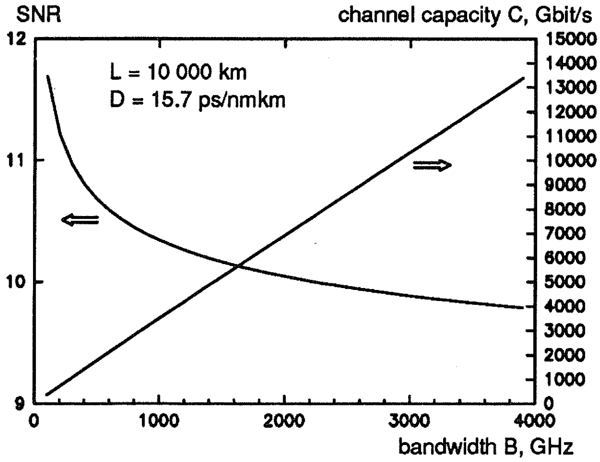

Figure 1: Signal-to-noise ratio and channel capacity versus bandwidth

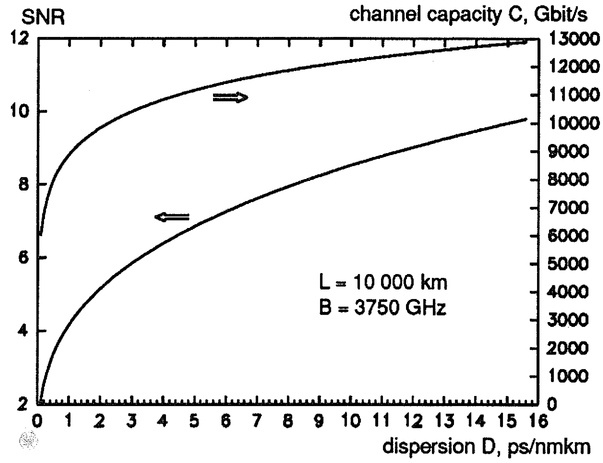

Figure 2: Signal-to-noise ratio and channel capacity versus dispersion

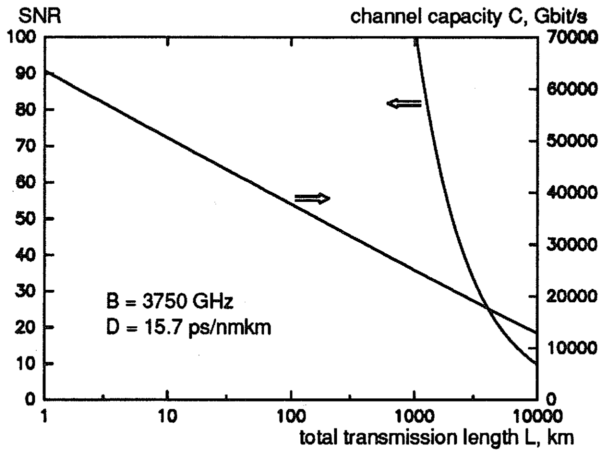

Figure 3: Signal-to-noise ratio and channel capacity versus transmission distance

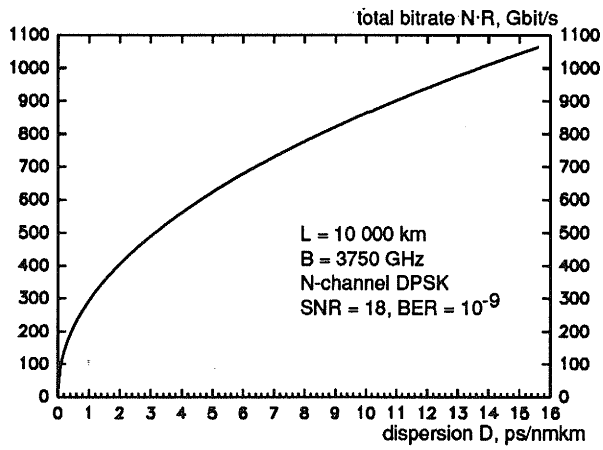

Figure 4: Total bitrate in a multi-channel MSK system versus dispersion

High bandwidth systems usually employ low dispersion fibers and for low dispersion ($b^2 < 2\pi$) we obtain:

$$X(\omega_c) = 16\gamma^2 \left(\frac{1}{2\pi}\right)^2 W^3 L^2 \pi^2 B^2 = 4\gamma^2 W^3 L^2 B^2 \tag{8}$$

With the same parameters as above, the same SNR_{opt} can be obtained in the low dispersion limit only by reduction of the bandwith B to:

$$B = 7.5\text{GHz} \quad \text{yielding } P_{\text{opt}} = B \cdot W_{\text{opt}} = 6.5\mu\text{W}, \quad \text{and thus } C = 25.76\text{Gbit/s}$$

These values can be obtained by single-channel systems, where an analysis in the time domain is probably more adequate, to describe the attainable bitrates. In this example the low dispersion limit $b^2 < 2\pi$ corresponds to $D = 0.2\text{ps/(nm}\cdot\text{km)}$ which is a value likely to occur in actual systems. Therefore, for real long length, high bandwidth transmission systems, considerably higher dispersions must be used.

4 Application to multi-channel MSK systems

In a more realistic system we have to build up our signal source by N channels, for example. In our example, each channel is MSK(minimum-shift-keying)-modulated and requires a bandwidth R equal to the bitrate R. The channels are equally distributed over our total bandwidth B. We define a factor of occupancy η of our band B by $\eta = N \cdot R/B$ and obtain consequently a reduced nonlinear noise density X':

$$X' = \frac{4\gamma^2 2 \cdot \eta^2 W^3 L}{\pi|\beta_2|} \left(1 + \ln\left(|\beta_2| L 2\pi B^2\right)\right) \tag{9}$$

The additional factor of 2 accounts for the correlation of pairs of spectral lines symmetric to the carrier in a MSK signal.
On the receiver side we wish to have a signal-to-noise-ratio SNR=18 in order to obtain a bit error rate of BER=10^{-9}.
As shown in Figure 4 a total capacity of up to $\eta B = NR =$ 1000Gbit/s can theoretically be achieved, if sufficient dispersion is present on the fiber ($D =$ 15.7ps/nmkm) and dispersion compensation is carried out at the receiver.

5 Other nonlinear effects

With the low spectral power density in our system considerations we are far below the Brillouin scattering threshold [7]. Due to stimulated Raman scattering spectral lines at higher frequencies will transfer power to spectral lines at lower frequencies [9]. The effective gain for the high frequency lines will thus be reduced with respect to the low frequency lines. This nonlinear Raman scattering effect may be compensated for by suitable tailoring the spectral gain characteristics of the EDFAs. For such an optimized system, only the Four-Photon-Mixing phenomena as discussed here, remain.

6 Conclusions

The presented results clearly show the reduction of nonlinear distortion in optical fibers by the linear fiber dispersion. Although dispersion shifted fibers are very useful in single channel systems, standard dispersion fibers with dispersion compensation after a long distance are superior with respect to high capacity multi-channel systems. The presented values for the channel capacity or the total bitrate are theoretical maximum values, which will be reduced by realistic amplifier spacings Δ_{OA}, amplifier excess noise, and other nonidealities ($n_{sp} > 1, \ldots$).

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