Chapter 9

Light-emitting diodes (LED)

Electro luminescence diodes and semiconductor are mainly manufactured as double heterostructure diodes lasers in optical telecommunication systems. In this structure the active region, where the recombination of charge carriers takes place, is surrounded by two layers with a larger band gap $W_G$. The name double heterostructure is due to the fact that both sides of the active region are surrounded with different semiconductor material (e.g. InGaAs-InP in Fig. 9.1).

![Figure 9.1: Example of a double heterostructure diode](image)

Fig. 9.1 shows such a diode. Basically, it is a p-n diode, which is operated in forward direction. The charge carriers recombine under radiation in the active region.

Fig. 9.2 shows the energy band diagram of a double heterostructure diode in forward direction during operation. A double heterostructure is characterized by a small active region with the band gap energy $W_{G2}$ which is much smaller than the band gap energy of the sandwich layers $W_{G1}$ and $W_{G3}$. In a pure pn-junction the inversion condition $W_{F_n} - W_{F_p} > W_G$ can only be reached for very high currents. The trick of the double-heterostructure is that both, the electrons in the conduction band as well as the holes in the valence band are limited by energy barriers of the structure. As a result, simultaneously very large concentrations of holes and electrons are gathered in the very thin active region $d$. This provides a high spontaneous and stimulated emission. The thickness of the active layer in LEDs is typically around $d \leq 1 \mu m$. In semiconductor lasers, this thickness can be reduced even to a few nanometers (i.e. a few atomic layers) (see also: quantum-well lasers in chapter HL-STRUK). Thus, even for low injected current densities, a high charge carrier concentration in the active region can be achieved.
9.1 Electro luminescence diodes

The recombination in electro luminescence diodes is mainly determined by the spontaneous emission. It is proportional to the product of electron density $n$ in the conduction band and hole density $p$ in the valence band.

$$R_{sp} \sim f_L (1 - f_V) \sim n \cdot p$$

(9.1)

More precisely:

$$R_{sp} = V \cdot B \cdot n \cdot p$$

(9.2)

where $B$ is the recombination coefficient in the magnitude of $10^{-10}$ cm$^3$/s.

Assuming that $n$ and $p$ are much higher than in the thermodynamic equilibrium, the spontaneous emission rate per volume is of the form (without current injection):

$$\frac{R_{sp}}{V} = -\frac{dn}{dt} = -\frac{dp}{dt} = B \cdot n \cdot p$$

(9.3)

The rate equations for the both carriers is given by

$$\frac{dp}{dt} = \frac{dn}{dt} = -\frac{R_{sp}}{V} + \frac{I}{e \cdot V}$$

(9.4)

where $I$ is the injection current. The dynamic modulation characteristics of an LED can be described by the spontaneous emission lifetime $\tau_{sp}$. If the spontaneous emission is e.g. defined as $R_{sp}/V = n/\tau_{sp}$, the charge carrier density $n$ decay, after switching off the injection current $I$ ($I = 0$), is given by:

$$n = n_0 \cdot \exp \left( -\frac{t}{\tau_{sp}} \right)$$

(9.5)

The maximum modulation frequency can thus be estimated by:

$$f_g \approx \frac{1}{2 \pi \cdot \tau_{sp}}$$

(9.6)
1. Undoped active region

For an undoped active region, the charge neutrality \( n = p \) applies and therefore:

\[
\frac{dn}{dt} = - \frac{R_{sp}}{V} = -B \cdot n^2 = -\frac{n}{\tau_{sp}} \tag{9.7}
\]

with

\[
\tau_{sp} = \frac{1}{B \cdot n} \tag{9.8}
\]

Hence, the mean lifetime decreases with increasing carrier injection. For a fast modulation a high carrier injection (or high \( n \)) is required.

Example: Assuming \( n = 3 \cdot 10^{18} \text{ cm}^{-3} \) and \( B = 2 \cdot 10^{-10} \text{ cm}^3 \text{ s}^{-1} \) the lifetime of the spontaneous emission is \( \tau_{sp} = 1.7 \text{ ns} \).

Such an LED is thus modulatable up to about 100 MHz.

2. Doped active region

In a doped active region \( n \) and \( p \) are not necessarily much larger than in the thermodynamic equilibrium. For this reason, Eq. (9.7) has to be modified.

(a) p-doping with the acceptor concentration \( N_A \)

\[
p \approx n + N_A \tag{9.9}
\]

\[
\frac{dn}{dt} = -B \cdot n \cdot p = -B \cdot n \cdot (n + N_A) \tag{9.10}
\]

\[
\Rightarrow \tau_{sp} = \frac{1}{B \cdot (n + N_A)} \tag{9.11}
\]

(b) n-doping with the donor concentration \( N_D \)

\[
n \approx p + N_D \tag{9.12}
\]

\[
\frac{dp}{dt} = -B \cdot p \cdot (p + N_D) \tag{9.13}
\]

\[
\Rightarrow \tau_{sp} = \frac{1}{B \cdot (p + N_D)} \tag{9.14}
\]

For \( N_D \gg p \) and \( N_A \gg n \), respectively, the mean lifetime of a doped active region becomes largely independent of the charge carrier concentration. With increasing doping, the LED can be modulated.
more quickly. This, however, increases the non-radiative recombination with the lifetime $\tau_{ns}$. At high non-radiative recombination, it is reasonable to define an effective lifetime $\tau_e$:

$$\frac{1}{\tau_e} = \frac{1}{\tau_{sp}} + \frac{1}{\tau_{ns}}$$  \hspace{1cm} (9.15)

in which for a high doping $\tau_e$ becomes relatively short. Therefore a high modulation frequency $f_g = \frac{1}{2\pi \tau_e}$ is feasible (at the cost of efficiency).

![Figure 9.3: Surface-emitting diode (Burrus-type LED)](image)

To ensure a good heat dissipation, it is important that the active region is located close to the heat sink. The current flow has to be limited to a spot size of the diameter $d_L$. In Fig. 9.3 for example the active region consists of GaAs where the heterostructures are made of GaAlAs on a substrate of GaAs. Since in this case, for example, the substrate is not transparent for the emission wavelength, a well is etched into the substrate in order to bring in the fiber (Burrus-type LED). The light emission of the LED, shown in Fig. 9.3, is perpendicular to the layer sequence. Such an LED is also called a surface-emitting LED.

Example: For a typical dimension with the data

- $d_L = 30 \mu m$  \hspace{1cm} Diameter of the light spot
- $d = 0.5 \mu m$  \hspace{1cm} Thickness of the active region
- $I = 100 mA$  \hspace{1cm} Injection current

$n$ and $\tau_e$ ($= \tau_{sp}$ should be determined by assuming $\tau_{ns} = \infty$).

The active volume is $V = \frac{\pi}{4} \cdot d^2_L \cdot d = 3.5 \cdot 10^{-10} cm^3$. Due to Eq. (9.4) and Eq. (9.7) and assuming $\tau_e = \tau_{sp}$ in steady state ($\frac{d}{dt} = 0$):

$$n = \frac{\tau_e \cdot I}{e \cdot V}$$  \hspace{1cm} (9.16)

According to Eq. (9.8), for an undoped region, Eq. (9.16) can be written as

$$n^2 = \frac{I}{e \cdot V \cdot B}$$  \hspace{1cm} (9.17)
With $B = 2 \cdot 10^{-10} \text{cm}^3 \cdot \text{s}$, $e = 1.6 \cdot 10^{-19} \text{As}$ and the assumed data above the charge carrier density results in $n = 3 \cdot 10^{18} \text{cm}^{-3}$. Therefore the effective mean lifetime is $\tau_e = 1.7 \text{ns}$ corresponding to the limiting frequency of approximately $f_g \approx 100 \text{MHz}$.

### 9.2 Optical power output

The efficiency of an LED - defined as the ratio of emitted optical power to electrical power input - is usually relatively low (in the order of a few percent).

Because of the high refractive index of the active region ($n \approx 3.5$) the largest part of the spontaneous emission is totally reflected and is not emitted out of the semiconductor. The non-radiative recombination also reduces the efficiency. Since the quantity of the non-radiative recombination depends on the doping, the power output of the diode also depends on the doping, leading to a dependency to the cutoff frequency $f_g$ of the diode (Fig. 9.4). Instead of the output power $P$ it is customary to declare the *radiance* $S_R$. It is defined as the output power per dihedral and solid angle unit and is of the dimension $\frac{\text{W}}{\text{cm}^2 \cdot \text{sr}}$ (sr = unit of the solid angle, actually dimensionless). The LED as surface-emitter has a *Lambertian radiation pattern* (circular radiation pattern in Fig. 9.3):

$$S(\gamma) = S_R \cos(\gamma) \quad (9.18)$$

The total emitted power is thus

$$P_0 = \frac{\pi}{4} \cdot d_L^2 \int S(\gamma) \, d\Omega \quad (9.19)$$

where the integration in Eq. (9.19) is determined by integrating over a half space.
Hence (see also Fig. 9.5)

\[
P_0 = \frac{\pi}{4} \cdot d_L^2 \cdot S_R \int_0^{\frac{\pi}{2}} 2\pi \sin(\gamma) \cos(\gamma) \, d\gamma = \frac{\pi^2}{4} \cdot d_L^2 \cdot S_R
\]  

(9.20)

Example: Assuming \( S_R = 100 \frac{W}{\text{cm}^2\text{sr}} \) and \( d_L = 50 \mu\text{m} \) the total emitted output power is \( P_0 = 6.2 \text{mW} \).

**9.3 Coupling of an LED to a step-index fiber**

Initially, it is assumed that the diameter of the LED light spot is greater than the diameter of the fiber \( (d_L > 2a) \). The LED and the fiber should be arranged as shown in Fig. 9.3. The numerical aperture of the fiber should be sufficiently small, so that a constant power density \( S(\gamma) \) can be assumed for all acceptance angles.

With

\[
\begin{align*}
\pi \cdot a^2 & \quad \text{Surface of the fiber end} \\
\pi \cdot A_N^2 & \quad \text{acceptance angle}
\end{align*}
\]

the power \( P_1 \) coupled into the step-index fiber is given by:

\[
P_1 = S_R \cdot \pi^2 \cdot a^2 \cdot A_N^2
\]

(9.21)

Example: \( S_R = 100 \frac{W}{\text{cm}^2\text{sr}} \), \( a = 25 \mu\text{m} \), \( A_N = 0.2 \)

\[
\Rightarrow \quad P_1 = 0.25 \text{mW}
\]

(9.22)

The coupling efficiency is determined by the ratio of the coupled power \( P_1 \) to the total emitted power \( P_0 \).

\[
\eta = \frac{P_1}{P_0} = \left( \frac{2a}{d_L} \right)^2 A_N^2 \quad \text{for} \quad 2a \leq d_L
\]

(9.23)

If the diameter of the fiber is larger than the diameter of the light spot \( (2a \geq d_L) \), the coupled power is of the form

\[
P_1 = S_R \left( \frac{\pi \cdot d_L}{2} \right)^2 A_N^2
\]

(9.24)
and by using Eq. (9.20) the coupling efficiency is

\[ \eta = \frac{P_1}{P_0} = A_N^2 \]  

(9.25)

The coupling efficiency from the LED into the fiber is therefore relatively low. Due to Eq. (9.25), for example, the coupling efficiency for \( A_N = 0.2 \) is thus only 4%.

To which extent the coupled power into the fiber can be increased by using suitable imaging according to Eq. (9.21) and (9.24)? We consider a common system of Fig. 9.6. In Fig. 9.6 assuming a lossless system

![Diagram](image)

Figure 9.6: Coupling of an LED with the surface \( F_1 \) into a fiber of the surface \( F_2 \) by means of an imaging system

the power \( P_{10} \) is defined as the power before the imaging system, while \( P_{20} \) is the power after the imaging system.

\[ P_{10} = P_{20} \]  

(9.26)

applies. Similar to the coupling of fibers (chapter KOP), the imaging equation is given by

\[ F_1 A_{N1}^2 = F_2 A_{N2}^2 \]  

(9.27)

Hence the radiance is of the form:

\[ S_{R1} = \frac{P_{10}}{\pi F_1 A_{N1}^2} = \frac{P_{20}}{\pi F_2 A_{N2}^2} = S_{R2} \]  

(9.28)

The radiance is conserved while passing through a lossless optical system. In particular, the radiance of e.g. an LED can not be increased. Therefore we can obtain:

1. For a given radiance of an LED the maximum coupling power is given by Eq. (9.21).

2. If the diameter of the fiber is smaller than the diameter of the luminous spot \( (2a < d_L) \), an imaging optic between LED and fiber brings no improvement.

3. If the diameter of the fiber is larger than the diameter of the luminous spot \( (2a > d_L) \), the coupled power can be increased maximal by the factor \( \left( \frac{2a}{\pi} \right)^2 \) using an enlarging imaging.

Eq. (9.21) can be generalized by using

\[ F_1 A_{N1}^2 = \frac{\lambda^2}{2\pi} M \]  

(9.29)
to \( (M - \text{number of eigenmodes or degrees of freedom, respectively}) \)

\[
P_1 = S_R \cdot \frac{\lambda^2}{2} \cdot M \quad (9.30)
\]

Example: The power to be coupled into a single-mode fiber with \( \lambda = 0.85 \mu \text{m} \), \( S_R = 100 \frac{\text{W}}{\text{cm}^2 \text{sr}} \) and \( M = 2 \) is \( P_1 = S_R \cdot \lambda^2 = 0.72 \mu \text{W} \).

Because of the low coupled power, the use of surface-emitting LEDs for optical communication systems is only useful in combination with multi-mode fibers.

If the LED is designed as an edge emitter, radiances of \( S_R > 1000 \frac{\text{W}}{\text{cm}^2 \text{sr}} \) can be achieved. The structure of an edge emitter is already very similar to a semiconductor laser (see Fig. 9.7).

Figure 9.7: Example of an edge-emitter